Erratum: Minimal renormalization without ε-expansion: Four-loop free energy in three dimensions for general n above and below T_c [Phys. Rev. E 67, 056115 (2003)]

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Recently it has been shown by Chen and Dohm [1] that, within a (d,n) universality class, two-scale factor universality in the form of our Eq. (11) is valid only for isotropic systems and systems with cubic symmetry. For anisotropic systems of noncubic symmetry in the same (d,n) universality class, our Eq. (11) must be replaced by Eq. (3) of Chen and Dohm [1].

On the right-hand side (rhs) of Eq. (91) the term $+(n-1)\frac{880}{3}\mathrm{Li}_2(-\frac{1}{3})$ should be added, thus the term $+\frac{880}{3}\mathrm{Li}_2(-\frac{1}{3})$ should be included in the curly brackets of Eq. (91).

At the end of Eq. (105) the term $+\frac{1}{8}X_{430}$ should read $-\frac{1}{8}X_{430}$.

On the rhs of Eq. (134) the second term should be multiplied by the factor $a_0^2/4$.

On the left-hand side (lhs) of Eq. (208) the upper limit $L-m$ of the second sum should read $4-m$.

Eq. (217) should read

$$f_3 = -8[96\zeta(3)(5n+22) + 8n^3 + 405n^2 + 4738n + 13776]. \tag{217}$$

In Eq. (218), the term $+\frac{1601680}{3}$ should be replaced by $+503544$.

In Eq. (242), the term $-2n+88$ should read $+2n-88$.

In Eq. (256), in the first line after Eq. (256) and in the first line of the second paragraph after Eq. (259), Φ_4 should be replaced by Φ_3 because the four-loop series of R_+ in Eq. (254) is of $O(\hat{u}_B^3)$, unlike the four-loop series of Sec.VII C which is of $O(\hat{u}_B^4)$. Thus, Eq. (256) should read

$$(R_\xi^+)^3 = (4\pi)^{-1}\mathrm{opt}_{\hat{u}_B}\left[\Phi_3\left(\{f_{+m}^{(R)}\},\hat{u}_B\right)\right]. \tag{256}$$

In analogy to Eq. (210), $\Phi_3(\{x_m\},\hat{u}_B)$ follows from Eq. (206) for $L=3$ as

$$\Phi_3(\{x_m\},\hat{u}_B) = x_0 + \frac{x_1}{2}\rho(\rho+1)\hat{u}_B + (2\rho-1)x_2\hat{u}_B^2 + x_3\hat{u}_B^3,$$

with $\rho = 1+\varepsilon/\omega$, in agreement with Eq. (24) of Kleinert and Van den Bossche [2]. For a given ρ, the optimal value of $\hat{u}_B^\star(\rho)$ is now determined by

$$0 \overset{!}{=} \left(\frac{\partial\,\Phi_3(\{f_m\},\hat{u}_B)}{\partial\,\hat{u}_B}\right)_{\hat{u}_B=\hat{u}_B^\star(\rho)} = \frac{f_1}{2}\rho(\rho+1) + 2(2\rho-1)f_2\hat{u}_B^\star + 3f_3(\hat{u}_B^\star)^2.$$

The condition (211) yields

$$\Phi_3(\{f_m\},\hat{u}_B^\star(\rho)) = -1,$$

which determines ρ. The analytic form of ρ as a function of n in $d=3$ dimensions is given in Eqs. (61)–(64) of Ref. [2]. Using these values of ρ in Eq. (256) yields

$$R_\xi^+ = 0.270, \quad n=1, \tag{257}$$

$$R_\xi^+ = 0.357, \quad n=2, \tag{258}$$

$$R_\xi^+ = 0.424, \quad n=3, \tag{259}$$

which corrects the original equations (257)–(259) We note, however, that the solid line in Fig. 4 remains unchanged.

Eqs. (B13) and (B14) should read

$$c_{Q1} = \frac{16}{27}(n+2)\left[247n + 3905 - 288(n+8)\mathrm{Li}_2(-\tfrac{1}{3}) - 24(n+8)\pi^2 + 4(43n+182)\ln\frac{4}{3}\right] + 64\pi^2(n+14)\left[C(n) - \widetilde{S}_2(1,n)\right], \tag{B13}$$

$$\begin{aligned} c_{Q2} = 32(n+2)\Bigg\{ & \frac{32}{3}(5n+22)\Bigg(4J_{1,1}^{(1)} - 2J_{2,1}^{(1)} + 4J_{3,1}^{(1)} + 3E_1 - E_1' + 4E_1'' + \frac{\pi^4}{160} + \frac{1}{12}\left[\mathrm{Li}_2(\tfrac{1}{3}) - \mathrm{Li}_2(\tfrac{1}{6}) - \frac{(\ln 2)^2}{2}\right]\Bigg) \\ & - \frac{8}{27}(173n+178)\left[\frac{1}{2}\left(\ln\frac{3}{4}\right)^2 + \mathrm{Li}_2(-\tfrac{1}{4}) + \mathrm{Li}_2(-\tfrac{2}{3})\right] \\ & + 32(n^2+6n+20)c_4 - \frac{\pi^2}{27}(137n^2 + 2384n + 9548) \\ & + \frac{2}{9}(797n+1542)\ln\frac{5}{3} \\ & - \frac{32}{27}(60n^2 + 869n + 3634)\mathrm{Li}_2(-\tfrac{1}{3}) \\ & + (3n^2 + 50n + 244)\zeta(3) \\ & + \frac{1}{27}(944n^2 + 7248n + 24544)\ln\frac{4}{3} + \frac{4001}{144}n^2 \\ & + \frac{3489713}{5832}n + \frac{9505097}{2916}\Bigg\} + 64\pi^2(4n^2 + 69n + 458)\left[C(n) - \widetilde{S}_2(1,n)\right]. \end{aligned} \tag{B14}$$

In Eq. (C4), the term $2\mathrm{Li}_2\left(-\frac{1}{3}\right)$ should read $2\mathrm{Li}_2\left(\frac{1}{3}\right)$, the term $+54\mathrm{Li}_2\left(\frac{1}{3}\right)$ should read $+54\mathrm{Li}_2\left(-\frac{1}{3}\right)$, and the term $+32(n+2)/(-2r_0')^2\, u_0^3/(4\pi)^4$ should be replaced by $-128(n+2)/(-2r_0')^2\, u_0^3/(4\pi)^4$.

In Eq. (C10) the overall minus sign is incorrect, thus the term $-n(n+2)/(4\pi)^4$ should read $n(n+2)/(4\pi)^4$.

In Eq. (C14), the term $2\mathrm{Li}_2\left(-\frac{1}{3}\right)$ should read $2\mathrm{Li}_2\left(\frac{1}{3}\right)$, the term $+54\mathrm{Li}_2\left(\frac{1}{3}\right)$ should read $+54\mathrm{Li}_2\left(-\frac{1}{3}\right)$, and an overall minus sign is missing, thus the factor $1/(4\pi)^3$ should read $-1/(4\pi)^3$, in agreement with Eq. (49) of Strösser *et al.* [3], which is written in a slightly different form.

In Eq. (C15) the term $-12(n-1)(n+2)\left[4c_2+42\mathrm{Li}_2\left(\frac{1}{3}\right)+21(\ln 3)^2\right]$ should read $-6(n-1)(n+2)\left[8c_2+42\mathrm{Li}_2\left(\frac{1}{3}\right)+21(\ln 3)^2\right]$, and the term $(584n^2+1135n+868)$ should read $(584n^2+1385n+434)$.

The second sentence after Eq. (C15) should be deleted.

In Eq. (D4), the term $+\frac{50565}{27}\ln 2$ in the third line should read $+\frac{50656}{27}\ln 2$; a minus sign is missing before the last term at the end of the fifth line of Eq. (D4), thus the term $-840\left[(\ln 3)^2+2\mathrm{Li}_2\left(\frac{1}{3}\right)\right]\frac{51200}{3}\mathrm{Li}_2\left(-\frac{1}{3}\right)$ should read $-840\left[(\ln 3)^2+2\mathrm{Li}_2\left(\frac{1}{3}\right)\right]-\frac{51200}{3}\mathrm{Li}_2\left(-\frac{1}{3}\right)$.

In Eq. (E1) the functional symbol "exp" should be inserted between $A(u(l_\pm),\varepsilon)$ and the integral.

In Eq. (E8), $\zeta_r(u')$ should read $\zeta_r(u'')$.

In the integrand of Eq. (E14), u' should be replaced by u''. In the second line after Eq. (E14), u' should be replaced by u''.

Equations (E17)–(E21) are applicable only to the case $\alpha<0$. In order to derive the singular part (183) of the free energy for $\alpha>0$ it is necessary to proceed from (133) in a way similar to (E7)–(E12). Thus the integral in (133) can be written as

$$\int_1^{l_\pm} B(u(l'))\left[\exp\int_{l_\pm}^{l'}(2\zeta_r-\varepsilon)\frac{dl''}{l''}\right]\frac{d1'}{l'} = \int_u^{u(l_\pm)}\tilde{f}(u',u(l_\pm))\tilde{G}(u',u(l_\pm))du',$$

with $\tilde{G}(u',u(l_\pm))=\partial\tilde{g}(u',u(l_\pm))/\partial u'$ where $\tilde{f}$ and $\tilde{g}$ are given by (E8) and (E11). Partial integration yields the leading term

$$\tilde{f}(u(l_\pm),u(l_\pm))\tilde{g}(u(l_\pm),u(l_\pm)) = -B(u(l_\pm))\nu/\alpha,$$

which, for $u(l_\pm)\to u^*$, leads to (183). It can be shown that the subleading terms $-\tilde{f}(u,u(l_\pm))\tilde{g}(u,u(l_\pm))$ and

$$-\int_u^{u(l_\pm)}\tilde{g}(u',u(l_\pm))\frac{\partial}{\partial u'}\tilde{f}(u',u(l_\pm))du'$$

behave asymptotically as $\sim\mathrm{const.}\, l_\pm^{\alpha/\nu}\sim\mathrm{const.}\,\xi_\pm^{-\alpha/\nu}$. Thus, together with the prefactor $\xi_\pm^{-d}$ in Eq. (133), these terms yield a regular contribution $\sim\mathrm{const.}\, t^{\alpha+\nu d}\sim t^2$ to the free energy.

None of the conclusions of our original paper are affected. All other equations of the original paper remain unchanged.

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[3] M. Strösser, S. A. Larin, and V. Dohm, Nucl. Phys. B **540[FS]**, 654 (1999); **549[FS]**, 668(E) (1999).